# Controlling Information Diffusion with Irrational Users

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***Abstract*—Understanding how information propagates over networks is critical to the development of online social networking. Different from existing works on modeling the rational behaviors in information diffusion, in this paper, we focus on the study of how to control the information propagation over networks through the use of irrational users. With the help of graphical evolutionary game theory, we analyze how the irrational users influence the rational neighbors and thus the whole rational networks. Simulation results verify our theoretic analysis, and show that with a few irrational users, the number of rational users adopting the forwarding strategy can be significantly increased, i.e., we are able to control the information diffusion with irrational users.**

***Index Terms*—Information diffusion, social media, graphical evolutionary game, irrational users.**

## I. Introduction

With recent development of social media, Internet of things, big data and many other new technologies, we witness the emergence of networks of crowd intelligence, where deeply connected individuals, enterprises and government agencies constantly interact with and influence each other. These new patterns of social interaction and industry operation and management have significant impact on our society and economy, sometimes detrimental. One example is the “salt panic” in China after the 2011 Tohuku Tsunami, where rumors circulated over internet and text messages caused “panic-shopping of salt”, “mob scenes at stores” and “5 to 10-fold jump of salt price” [1] [2]. Therefore, it is of crucial importance to model how information propagates over such networks of crowd intelligence, to understand how such information propagation influence users’ decisions and the entire networks, and to design effective mechanisms to manage information propagation over networks.

In the literature, many works on information diffusion have been proposed, which are either using machine learning approaches to make macroscopic inference and prediction, or utilizing the game theory to analyze the microscopic interactions among users. Yang and Leskovec proposed to identify the diffusion dynamics pattern of the online contents with a clustering algorithm [3]. Pinto et al. proposed to predict the future information diffusion with the early diffusion data [4], and the community structure is also utilized to further improve the prediction performance of viral memes [5]. The impact of social network cluster structure on the diffusion behaviors was investigated in [6]. The main drawback of machine learning approaches is the lack of understanding the microscopic interaction among users, which is the actual reason that information diffusion happens.

To understand the microscopic interaction among users, game theory is often utilized. In [7] [8], the game-theoretic mechanisms were developed to analyze the competitive contagions in networks. Morris studied the conditions for global contagion of behaviors based on the assumption that each user played the best response to the population strategies [9]. Evolutionary game-theoretic frameworks were utilized to model the user’s interactions during the information diffusion process [10] [11] [12]. It was shown in [10] [11] [12] that with the evolutionary game-theoretic frameworks, the theoretically derived dynamics fit well with the real-world information diffusion dynamics and some future diffusion dynamics can even be predicted.

While the existing game-theoretic approaches achieve promising performance in modelling how information diffuses over networks, most of them rely on the assumptions that all users are rational and follow the natural selection updating rules to update their strategies. However, in reality, irrational user behaviors are often observed and people are actually utilizing the irrational users to regulate and/or control the information diffusion over networks. Examples include rumor spreading and product advertisement. In both examples, there are some irrational users intentionally forwarding information, either rumors or product information, to their neighbors to spread information. Notice that these irrational users are often paid to do so, and thus their objectives are different from the rational users. Obviously, the existence of the irrational users will significantly impact the information propagation, and the study of how the irrational behaviors affect the information diffusion is critical to the further development of online social networking. In this paper, we exploit the graphical evolutionary game theoretical framework to analyze the influence of the irrational behaviors, with the objective to control the information diffusion among rational users through the use of irrational users.

The rest of this paper is organized as follows. In section 2, we describe the system model in detail. Then, we analyze the evolutionary dynamics in section 3. The simulation results are discussed in section 4 and conclusions are drawn in section 5.

## II. System Model

The main focus of this paper is to study how to control the information propagation through the use of irrational users. We tackle this problem with the graphical evolutionary game formulation as in [11]. In such a formulation, the players are the users in a network, the strategies are to forward the received information $S_f$ or not forward $S_n$, and the payoff matrix depends on the interactions among users, which can be written as follows

$$U = \begin{array}{c} \\ S_f \\ S_n \end{array} \begin{array}{c} \begin{array}{cc} S_f & S_n \end{array} \\ \begin{pmatrix} u_{ff} & u_{fn} \\ u_{fn} & u_{nn} \end{pmatrix} \end{array}, \tag{1}$$

where $u_{ff}$, $u_{fn}$ and $u_{nn}$ represent the payoff when two users both with strategy $S_f$ interact, the payoff when a user with strategy $S_f$ meets a user with strategy $S_n$, and the payoff when two users both with strategy $S_n$ interact, respectively. The final utility of a player is referred as fitness [11] as follows

$$\Psi = (1-\alpha)B + \alpha U, \tag{2}$$

where $B$ is the baseline fitness representing the player's inherent property, $U$ is the payoff as in (1), and $\alpha$ is the selection intensity parameter determining the relative contribution of the game to fitness. Similar to [10], we consider the weak selection scenario, i.e., $\alpha \ll 1$.

We consider a regular network with a total number of $N$ rational users in the network. These rational users update their strategies, i.e., to forward $S_f$ or not to forward $S_n$, according to the natural selection updating rule. Besides the $N$ rational users, we assume that there are $L$ irrational users which are controlled by a specific organization, and the irrational users always adopt $S_f$ and never change. Without loss of generality, in this paper we focus on the birth-death (BD) natural selection updating rule. Other updating rules such as death-birth (DB) and imitation (IM) can be analyzed similarly.

The $N$ rational users can be classified into two groups: the first group of rational users are directly connected to the irrational users and the second group of rational users are not directly connected to the irrational users. We assume that the number of first group of rational users is $M$, and the probability that a rational user is directly connected to $h$ irrational users is $g(h)$. For each irrational user, the probability that it is connected to $l$ rational users is $f(l)$. Note that the irrational users influence the first group of rational users directly, while the second group of rational users are indirectly influenced through propagation with natural selection updating rule.

## III. Evolutionary Dynamics

Let $x_f$ and $x_n$ be the proportion of rational users with strategy $S_f$ and $S_n$, respectively. Denote $x_{f|f}$ and $x_{f|n}$ be the proportion of the neighbors adopting strategy $S_f$ given the user is with strategy $S_f$ and $S_n$, respectively. Similar to [11], the optimal $x^*_{f|f}$ and $x^*_{f|n}$ can be expressed as functions of $x_f$ as follows

$$\begin{cases} x^*_{f|n} = \frac{k-2}{k-1}x_f, \\ x^*_{f|f} = \frac{k-2}{k-1}x_f + \frac{1}{k-1}, \end{cases} \tag{7}$$

where $k$ is the degree of each user in the regular network.

Let $\Psi^i_f$ and $\Psi^i_n$ be the fitness of the $i^{th}$ group of rational users when adopting strategy $S_f$ and $S_n$, respectively. According to (2), $\Psi^i_f$ and $\Psi^i_n$ can be written as follows

$$\begin{cases} \Psi^1_f = 1-\alpha+\alpha\left[(k_f+h)\,u_{ff} + (k-k_f)\,u_{fn}\right], \\ \Psi^1_n = 1-\alpha+\alpha\left[(k_f+h)\,u_{fn} + (k-k_f)\,u_{nn}\right], \end{cases} \tag{8}$$

and

$$\begin{cases} \Psi^2_f = 1-\alpha+\alpha\left[k_f u_{ff} + (k-k_f)\,u_{fn}\right], \\ \Psi^2_n = 1-\alpha+\alpha\left[k_f u_{fn} + (k-k_f)\,u_{nn}\right], \end{cases} \tag{9}$$

where $k_f$ is the number of rational neighbors adopting strategy $S_f$, and $h$ is the number of irrational neighbors.

The proportion of rational users adopting strategy $S_f$ evolves either when a rational user's strategy is updated from $S_n$ to $S_f$ or when a rational user's strategy is updated from $S_f$ to $S_n$. In the following, we derive the occurrence probability of these two scenarios respectively. Specifically, according to the BD updating rule, the first scenario occurs when a user with strategy $S_f$ is chosen for reproduction and one of the rational neighbors, which was adopting strategy $S_n$, replicates the strategy $S_f$. Note that the user with strategy $S_f$ can be chosen from the first group of rational users, the second group of rational users, or the irrational users. Therefore, the occurrence probability of the first scenario can be written as follows

$$\begin{aligned} \Delta x^+_f = & \frac{N-M}{N+L}\sum_{k_f=0}^{k}\frac{k!}{k_f!(k-k_f)!}x^{k_f}_{f|f}(1-x_{f|f})^{k-k_f}\frac{x_f\Psi^2_f}{\bar{\Psi}}\frac{k-k_f}{k} \\ & +\sum_{h=1}^{L}g(h)\frac{M}{N+L}\sum_{k_f=0}^{k}\frac{k!}{k_f!(k-k_f)!}x^{k_f}_{f|f}(1-x_{f|f})^{k-k_f}\frac{x_f\Psi^1_f}{\bar{\Psi}}\frac{k-k_f}{k+h} \\ & +\sum_{l=1}^{M}f(l)\frac{L}{N+L}\sum_{l_f=0}^{l}\frac{l!}{l_f!(l-l_f)!}x^{l_f}_{f|f}(1-x_{f|f})^{l-l_f}\frac{\Psi^2_f}{\bar{\Psi}}\frac{l-l_f}{l} \end{aligned} \tag{10}$$

Similarly, the second scenario occurs when a user with strategy $S_n$ is chosen for reproduction and one of the rational neighbors, which was adopting strategy $S_f$, replicates the strategy $S_n$. Note that the user with strategy $S_n$ can only be chosen either from the first group of rational users or the second group of rational users. Therefore, the occurrence probability of the second scenario can be written as follows

$$\begin{aligned} \Delta x^-_f = & \frac{N-M}{N+L}\sum_{k_f=0}^{k}\frac{k!}{k_f!(k-k_f)!}x^{k_f}_{f|n}(1-x_{f|n})^{k-k_f}\frac{(1-x_f)\Psi^2_n}{\bar{\Psi}}\frac{k_f}{k} \\ & +\sum_{h=1}^{L}g(h)\frac{M}{N+L}\sum_{k_f=0}^{k}\frac{k!}{k_f!(k-k_f)!}x^{k_f}_{f|n}(1-x_{f|n})^{k-k_f} \\ & \times\frac{(1-x_f)\Psi^1_n}{\bar{\Psi}}\frac{k_f}{k+h} \end{aligned} \tag{11}$$

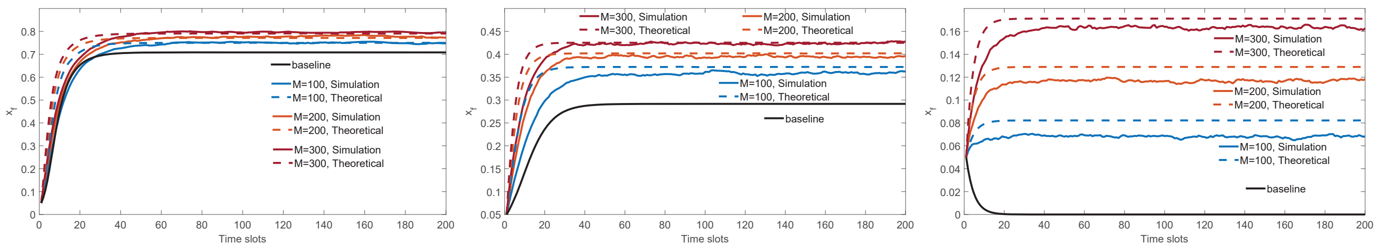
$$
\begin{aligned}
\dot{x}_f &= \Delta x_f^+ - \Delta x_f^- = \frac{N-M}{N+L}\frac{\alpha x_f(1-x_{f|f})(k-1)}{\bar{\Psi}}\left[\left(u_{ff}-u_{fn}\right)x_{f|f}-\left(u_{nn}-u_{fn}\right)\left(1-x_{f|n}\right)\right] \\
&+ \sum_{h=1}^{L} g(h)\frac{M}{N+L}\frac{k}{k+h}\frac{\alpha x_f(1-x_{f|f})}{\bar{\Psi}}\left\{(k-1)\left[\left(u_{ff}-u_{fn}\right)x_{f|f}-\left(u_{nn}-u_{fn}\right)\left(1-x_{f|n}\right)\right]+h\left(u_{ff}-u_{fn}\right)\right\} \\
&+ \sum_{l=1}^{M} f(l)\frac{L}{N+L}\frac{(1-x_{f|f})}{\bar{\Psi}}\left[1-\alpha+\alpha l u_{fn}+\alpha\left(u_{ff}-u_{fn}\right)(l-1)x_{f|f}\right] \\
&= \frac{k-2}{(k-1)(N+L)\bar{\Psi}}(1-x_f)(ax_f^2+bx_f+c). \qquad (3)
\end{aligned}
$$

where

$$
a = \alpha(k-2)(u_{ff}-2u_{fn}+u_{nn})\left[N-M+M\sum_{h=1}^{L}g(h)\frac{k}{k+h}\right]. \qquad (4)
$$

$$
\begin{aligned}
b &= \alpha\left[u_{ff}+(k-2)u_{fn}-(k-1)u_{nn}\right]\left[N-M+M\sum_{h=1}^{L}g(h)\frac{k}{k+h}\right]+\alpha M\left[\sum_{h=1}^{L}g(h)\frac{kh}{k+h}\right](u_{ff}-u_{fn}) \\
&+ \alpha L(u_{ff}-u_{fn})\left[\sum_{l=1}^{M}f(l)l-1\right]\frac{k-2}{k-1}. \qquad (5)
\end{aligned}
$$

$$
c = L\left[1-\alpha+\alpha u_{fn}\sum_{l=1}^{M}f(l)l+\alpha(u_{ff}-u_{fn})\frac{\sum_{l=1}^{M}f(l)l-1}{k-1}\right]. \qquad (6)
$$




(a) Payoff matrix PM1. (b) Payoff matrix PM2. (c) Payoff matrix PM3.

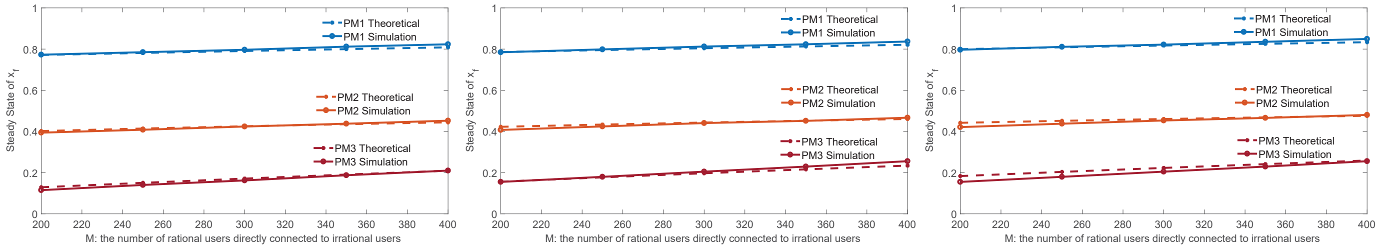
Fig. 1. The evolution process of $x_f$ with $L = 10$ at different $M$ and payoff matrices.

Combining (10) and (11), the evolution dynamics of the proportion of users adopting strategy $S_f$, i.e., $x_f$, can be derived as shown in (3). From (3), we can see that by setting $\dot{x}_f = 0$, there are three possible evolutionary stable strategies (ESSs), i.e., $x_f^* = 1$ and the other two ESSs are the solutions to $ax_f^2 + bx_f + c = 0$. The parameters $a$, $b$ and $c$ are shown in (4), (5) and (6), respectively. It is worth pointing out that the parameters $a$, $b$, and $c$ here not only depend on the payoff matrix, but also $L$, $M$, $g(h)$, as well as $f(l)$, which is different from those in [11]. On the other hand, when there is no irrational user, i.e., $L = 0$, then the parameter $c = 0$ and the parameters $a$ and $b$ reduce back to those in [11].

## IV. Simulation Results

In this section, we conduct simulations on synthetic networks to evaluate the proposed framework. We construct the whole network based on the random regular networks, which is consist of 1000 rational users with degree 10, i.e., 10 rational neighbors are randomly connected to for each rational user. Then, $M$ rational users are randomly selected to connect with $L$ irrational users according the homogeneous probability distribution $g(h)$. Without loss of generality, in this paper, we set $g(h)$ to the uniform distribution from 1 to 5, i.e., each of the selected rational users has the same probability to connect with one to five irrational users randomly. The weak selection parameter $\alpha$ is set to be 0.1. For each simulation, 10 graphs are randomly generated and 30 simulation runs are conducted for each graph. At the beginning, 5% rational users are randomly selected to adopt the $S_f$, and the BD updating rule is repeated until the whole networks reach the stable states in each simulation run. We consider three payoff matrices in our simulations:

- PM1: $u_{ff}$ = 0.6, $u_{fn}$ = 0.9, $u_{nn}$ = 0.3;
- PM2: $u_{ff}$ = 0.3, $u_{fn}$ = 0.9, $u_{nn}$ = 0.6;




(a) The scenario with $L = 10$. (b) The scenario with $L = 20$. (c) The scenario with $L = 30$.

Fig. 2. The steady states of $x_f$ versus $L$, $M$, and payoff matrices.

- PM3: $u_{ff}$ = 0.3, $u_{fn}$ = 0.6, $u_{nn}$ = 0.9.

In the first simulation, we fix $L = 10$ and vary $M$ from 100 to 300 to study its influence on the evolution process of strategy $S_f$ with respect to different payoff matrices. The results are shown in Fig. 1. We can see that the theoretical results match fairly well with the simulation results, where the gap is smaller than 0.02. We also plot the results where no irrational users exist as the baseline in Fig. 1 with black solid line. Comparing the result of different $M$ with the baseline, we can see that the $x_f$ increases faster then become stable and the stable state is larger as $M$ grows. Under the scenario with the payoff matrix PM3, the information which cannot spread out with the baseline now can diffuse to some extent with the existence of irrational users. It is consistent with the common sense that the rational users will stimulate the propagation of the information, and the more rational users that irrational users connect with, the wider the information will spread.

We also illustrate the stable states of $x_f$ with respect to different $L$, $M$ and payoff matrices. The results are shown in Fig. 2. We can again see that the theoretical results match well with the simulation results. Both increasing $M$ and $L$ will lead to the increase of $x_f$, which shows that more irrational users and connected edges between irrational users and rational users will both result in the boost of the information diffusion. With Fig. 1 and Fig. 2, we can observe that with $L = 10$ irrational users, the stable states of $x_f$ can increase about 0.15 when the number of first group ration users $M = 300$ under different payoff matrices.

## V. Conclusions

In this paper, we investigated how to control the information diffusion over networks through the use of the irrational users. Our theoretic analysis and simulation results show that with a few irrational users, the number of rational users adopting the forwarding strategy can be significantly increased. Moreover, we also observed that more irrational users and connected edges between irrational users and rational users can lead to wider information propagation.

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